The United States is a nation of debtors. By the end of 2007, total debt outstanding by households, businesses, state and local governments, and the federal government added up to $31.2 trillion. The domestic financial sector accounted for half of this total, or $15.8 trillion.

The size of the debt market is quite large. Indeed, it exceeds both the U.S. GDP in 2007 ($13.8 trillion) and the equity market value of all domestic corporations ($15.5 trillion). The primary risk of all this debt is credit risk, or the risk of default. The current credit crisis demonstrates how shifts in credit spreads and market liquidity can also significantly impact debt values. Although these alternative factors are important for understanding debt markets, we will focus here only on default risk.

Investors measure default risk in many different ways, and there have been important recent innovations in this regard. The state of the art in assessing corporate credit risk is based on one of three approaches: 1) the Merton distance-to-default measure, 2) the reduced-form approach, and 3) credit ratings. We will compare and contrast these three approaches, showing that the reduced-form approach is preferred because of its generality, flexibility, and superior forecasting ability.

Merton’s Distance-to-Default

For more than three decades, a common approach used to measure a firm’s default probability has been the so-called distance-to-default. This measure is based on the pioneering work of Robert C. Merton (1974), who applied contingent claims analysis to the pricing of corporate debt.

The idea is that creditors and equity holders jointly own the firm, but they do not share equally the risks of large increases or decreases in the firm’s value. Creditors have a fixed claim to the face value of the debt, but equity holders’ limited liability restricts them to a claim only on the firm’s upside potential. If a large firm’s value deteriorates to a point at which the firm’s asset value is less than the face value of the debt at maturity, the firm defaults and it belongs to the creditors. The debt holders lose part of their claims, but the equity holders lose everything. On the other hand, if the firm’s asset value is more than the face value of the debt, the equity holders receive the residual value of the firm over and above the debt’s face value.

In effect, equity holders hold a call option on the firm’s asset value with a strike price equal to the face value of the debt. Meanwhile, the debt holders own risk-free debt plus a short put option on the firm’s asset value. A call option is a financial security that gives its owner the right to purchase an asset at a predetermined price, called the strike price, on a predetermined date, called the maturity date. A put option is similar, except that it gives the right to sell, and not the right to purchase. Both types of options are widely traded on equities, foreign currencies, and interest rates. And both types of options have been modeled extensively using the tools of finance.
Making the analogy between equity and a call option on the firm’s asset value enables additional insights to be obtained via the use of these financial engineering tools.

Using this options analogy, it is easy to show that a firm’s probability of default depends primarily on two factors: 1) the size of the firm’s asset value relative to the face value of debt—if the debt/equity ratio is very high, default is likely; and 2) how volatile the firm’s asset value is—if the firm is very risky, default is more likely because the risk of a sudden deterioration in asset value is high. The firm’s one-year default probability therefore depends on the difference between the firm’s asset value and the face value of debt, where this difference is measured in standard deviations of the firm’s asset value. This is the distance-to-default measure expressed as the number of standard deviations the firm’s asset value has to drop before default occurs. The larger the number of standard deviations, the smaller is the probability of default.

The approach’s simplicity and intuitive appeal explain why it is the basis for commercially available corporate default probabilities offered by firms like Moody’s/KMV, which first popularized the concept, and Kamakura Corporation. This approach is also often referred to as the structural approach because the default probability is directly based on a model for the liability structure of the firm.

Unfortunately, the simplicity of the structural approach is both a good thing and a bad thing. It is good because it is easy to understand. It is bad, however, because this simplicity implies that the model has too few parameters to match market realities. Indeed, the distance-to-default has only two key inputs: the firm’s leverage ratio and the volatility of the firm’s asset value.

This theory has another difficulty. For simplicity of implementation, it assumes that the firm’s future asset value follows a lognormal distribution. This implies a specific linkage between the firm’s distance-to-default measure and the default probability: A distance-to-default of three standard deviations implies a default probability of close to 0.1%, and at four standard deviations the default probability is practically zero.

This linkage results in a relationship between distance-to-default and default probabilities that is historically inaccurate. Kamakura Corporation has found, for example, that the actual default rate among companies with a theoretical Merton default probability of zero is actually 0.20%. Looking at the other side of the distribution of default probabilities, the actual default rate among companies with theoretical Merton default probabilities over 90% is actually no more than 10%.

In order to better match historical defaults for practical application, the theoretical Merton default probabilities are necessarily modified. The theoretical default probabilities are mapped to empirical defaults, typically using a logistic regression. A logistic regression is ordinary linear regression where the independent variable is transformed to guarantee that it always lies between zero and one. This transformation of the independent variable is needed because default probabilities lie between zero and one.

Most analysts do this mapping of theoretical to empirical default probabilities so that the ranking of companies by credit riskiness is not changed; the mapping is done monotonically, when measured by either the “ROC accuracy ratio” or the “cumulative accuracy profile.” Based on these measures, the accuracy of the default model is not changed by the mapping process. However, the mapping does change the relationship between actual and expected defaults.

A final concern about the distance-to-default measure is that it appears less applicable to retail and small business clients. This is because, as commonly implemented, the distance-to-default measure requires the use of traded equity prices. Retail and small business clients would not have publicly traded equity. One vendor’s early attempt to modify and use Merton’s model for unlisted companies was ended after a decade without commercial acceptance. To our knowledge, no commercial product using the Merton model has ever been employed for retail risk assessment.
Reduced-Form Default Probabilities

An increasingly popular approach to measuring default probability is the reduced-form model. The reduced-form model is more flexible than Merton’s structural model because it allows for more factors to affect the probability of default. This approach has its origins in Merton (1976), who used a default intensity to value put and call options on a firm that may go bankrupt at any point in time, suddenly driving the stock price to zero. However, at that time, Merton was not interested in analyzing credit risk, but rather in valuing options on stocks whose prices exhibit large jumps. About 15 years later, applying this approach to credit risk, Jarrow and Turnbull (1992, 1995) introduced the reduced-form model.

The idea of the reduced-form approach is straightforward; corporate default may be triggered by many different factors, and default may happen at any point in time. For example, a firm that has no debt outstanding may not have enough cash to make salary payments to its staff and so cause a default. The Merton model does not allow the firm to default because it has no debt. Other examples of cash-flow-induced defaults include a firm that runs out of cash to make office lease payments or an unprofitable firm unable to raise additional capital to offset its losses. A firm that needs cash may find that the market for its assets has suddenly shut down (for example, the CDO market in 2007) or a firm that has borrowed too much may run out of funds for repayment. In addition, the reduced-form model allows the firm’s debt/equity ratio and other important firm liabilities to change across time. This is not true for Merton’s model.

As just explained, the reduced-form approach can incorporate many different inputs, including the inputs from the Merton approach. As such, it can never be less accurate than the Merton approach. In theory, this is a distinct advantage. The reduced-form approach chooses the inputs by estimating their relative importance in fitting historical defaults and so generates better predictions of future defaults. The reduced-form approach is more general because, unlike the structural approach, it does not assume a particular model of the firm’s liability structure. It therefore does not limit the model’s inputs to only the firm’s leverage and volatility. The reduced-form probability of default is calculated using all available information at a given point in time.

Another advantage of the reduced-form approach is that it can be used to estimate the probability of default not only over the next month but over any period of time—the next year or even the next five years. Creditors care about these long-horizon default probabilities because they can be used to help predict financial distress and avoid related investment losses. For example, it would have been helpful to know that Countrywide’s default probability was high enough to cause financial distress before its equity value dropped dramatically in the fall of 2007.

In a recent paper, Campbell, Hilscher, and Szilagyi (forthcoming) agree that default probabilities should be estimated over long horizons: “Over a short horizon, it should not be surprising that the recent return on a firm’s equity is a powerful predictor, but this may not be very useful information if it is relevant only in the extremely short run, just as it would not be useful to predict a heart attack by observing a person dropping to the floor clutching his chest.”

Comparing Structural and Reduced-Form Models

One way to evaluate the accuracy of these two approaches is to ask which approach, the reduced-form or the distance-to-default, better predicts historical defaults. Two recent studies, by Campbell, Hilscher, and Szilagyi (forthcoming) and Bharath and Shumway (forthcoming), show that the reduced-form approach has been better able to predict U.S. corporate defaults. Both studies also find several additional variables that increase the default-prediction accuracy of the Moody’s/KMV method of using only the distance-to-default. This result is not surprising. Simply put, both studies find that additional measures of firm characteristics add predictive value. Commercially, the Kamakura Corporation has also reported similar results with its reduced-form and Merton models in its technical guides for each of the four versions of the models.

Given the high predictive accuracy of reduced-form models and the ability to continually update them, it is surprising that so many practitioners in the financial markets are still using mainly the slow-moving distance-to-default-based default probabilities.

Credit Ratings

All of this brings us to the third, and probably most widely used, default risk measure of publicly traded debt securities: the bond’s credit rating. A credit rating is simply an ordinal ranking of companies by credit risk, typically using 20 rating categories. These groups could be labeled from 1 to 20 or from CC to AAA.

The default probabilities associated with each rating category are unknown, although they can be subsequently estimated. The implication is that all firms within a given rating category have the same default probability.
In contrast, in a reduced-form or structural model, each company in a given rating category would have a different default probability. Of course, the hope is that the firms falling into a particular rating category would have similar default probabilities as measured by these other techniques, but this need not be the case.

Rating agencies typically publish historical default statistics by rating category. The assumption implicit in these statistics is that the default probabilities are constant across time. Indeed, a common practice in modeling defaults is to use the historical one-year default rate for a given rating—say, CCC—across time despite the fact that the historical CCC default rate has varied from zero to 44% over the U.S. experience. The economic implication of the constant default rate assumption is that there is no business cycle. Of course, this is a fatally flawed assumption in light of U.S. economic history.

A further complication in rating categories is the lack of a horizon associated with a given rating. For a short-term horizon (say, one month), a BBB company in a good economy has nearly the same default risk as an AAA company: zero. In the longer term (a year or more), this is certainly not the case, if the ratings have been assigned correctly.

There is another aspect of the rating process that lowers its accuracy: The rating agencies are reluctant to change a rating “too often.” If a company’s risk suddenly rises and then falls 30 days later, the rating agencies, feeling their clients are opposed to a rating that falls and returns to its original level so quickly, will wait until they think there is no risk of a reversal before assigning a rating change. This wait-and-see approach is a reason why reduced-form models outperform agency ratings as predictors of default across all measured time horizons. Another reason is that the size of the company is the single most statistically significant factor in determining a company’s rating. Since company size is also used in estimating reduced-form default probabilities, further consideration of the rating process will not increase the accuracy of the default probabilities.

The imprecision of credit ratings has exacerbated the currently troubled credit environment. Many institutions have policies that restrict investment to either only AAA debt or only investment-grade debt. Historically, it was very difficult to receive an AAA rating from one of the major rating agencies, such as Standard & Poor’s, Moody’s, or Fitch. However, over the last few years, senior tranches of repackaged portfolios of debt securities (CDOs) have often received AAA ratings even though the constituent parts were often of much lower credit quality.

The Financial Times reported on December 18, 2007, that these high ratings were seriously flawed, and they were suddenly and severely downgraded in one abrupt adjustment. As a result, many investors were forced by their own investment policies to dump these troubled securities on the market in a manner guaranteed to result in poor execution of the trades. Consequently, many investors are shifting to an investment policy that relies on independent valuations and independent default probabilities in order to move more skillfully and quickly as an asset becomes troubled.

Of course, agency ratings are not available for retail borrowers, for small business borrowers, and for many listed corporations. But enterprise risk management depends on estimates of their default risk as well. To use the credit-rating approach, internal ratings need to be “mapped” to agency ratings in a consistent manner. This is often done by equalizing default rates across internal-rating and credit-rating categories, but there is still a problem. If CCC annual default rates averaged 27% but have varied from 0% to 44%, which default rate in this range should be used to map to an equivalent internal rating? Shouldn’t the business cycle be considered? Regardless of the approach used, the result is often a highly arbitrary process.

Conclusions
Best practice in risk management requires continuous improvement and perfect transparency. Risk managers are seeking an enterprise-wide ability to manage risk as the default probabilities across all borrowers—from retail to corporate to sovereigns—rise and fall with the macroeconomic factors driving the business cycle.

Financial institutions are now turning away from company-specific over-the-counter derivatives like credit default swaps to hedge these risks. Instead, they are turning to macro factor risk hedges using derivatives on interest rates, stock price indices, foreign exchange rates, and home price futures. Only a reduced-form approach to credit risk modeling allows a risk manager to achieve this objective with the essential speed and accuracy required.

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Notes
1. Source is the Board of Governors of the Federal Reserve System, flow of funds statistics.

Many investors are shifting to an investment policy that relies on independent valuations and independent default probabilities in order to move more skillfully and quickly as an asset becomes troubled.
2 For more on this point, see van Deventer, Li, and Wang (2006).

3 The Merton approach, by contrast, initially assumed that default occurs only at the end of a single period. Recently, Merton models have been generalized to allow defaults across time by employing the insights of Black and Cox (1976).


5 This constraint was stated clearly in a presentation by Moody’s Investors Service at a conference sponsored by PRMIA and DePaul University, Chicago Mercantile Exchange, February 28, 2008.

6 As reported by van Deventer, Li, and Wang (2006).


References


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