Dynamic volatility regulation of financial institutions*

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Abstract

Unlike non-financial firms, financial institutions are often heavily regulated to prevent bankruptcies and negative spillovers. A main regulatory tool is risk-based capital requirements. To reflect this reality, we have developed a model that allows for dynamically updated asset risk, in contrast to standard contingent claim models that assume constant volatility. Regulators impose a decrease in asset volatility when the capital cushion becomes small, thereby reducing the risk of distress. As a result, the dynamics of credit risk of financial institutions are different from unregulated firms, implying different credit spreads, transition matrices, ratings, valuation of liabilities, and cost of deposit insurance.

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1. Introduction

Financial intermediaries (like banks) are usually heavily leveraged relative to non-financial firms. High leverage creates credit risk, which can endanger the whole economic system. This is the main reason for strict control of banks (and many other financial intermediaries). The risk control is implemented typically through capital requirements that force banks to increase capital or to decrease risk as soon as there are first signs of financial distress.

Regulatory control of bank asset risk is now one of the main tools for banking regulation. Following the financial crisis of 2008, policymakers have adopted macroprudential policy and conduct stress tests in the U.S. and the EU (Berger and Demirgüç-Kunt 2021; Wachtel 2013). Stress tests evaluate whether banks have sufficient capital to continue lending and engaging in other banking functions in simulated adverse future scenarios (Gofman 2017; Goldstein and Sapra 2014; Goldstein and Leitner 2018; Glasserman and Young 2015). Institutions that fail the tests are generally required to take actions that either lead to a lower leverage by converting debt to equity (Flannery 2002; Raviv 2004) or to a lower level of risk by changing the composition of their assets (Berger and Demirgüç-Kunt 2021; Hilscher, Landskroner and Raviv 2021).

The literature on risk and regulation analyzes the effect of debt instruments that are converted to capital in times of financial distress (for example Hilscher and Raviv 2014; Shiller 2010; Glasserman and Nouri 2016; Martynova and Perrotti 2018), but it does not account for dynamic changes in volatility. These models assume that regulators perform periodic audits, and in case of financial distress, banks are either liquidated or reorganized based on the forbearance policy established by the regulators (Merton 1977, 1978; Ronn and Verma 1986, Marcus and Shaked 1984). The practice of reducing the level of asset risk in times of distress by changing a bank asset composition is not explored.

To bridge this gap, we propose a framework that extends the classical Merton (1974, 1977) contingent claim model, but also incorporates dynamic volatility contingent upon the financial institution's severity of financial distress. Unlike the traditional model, which assumes a constant volatility of assets, our framework introduces a mechanism where the regulator intervenes when the bank's leverage surpasses a certain threshold. This intervention aims to restrict the level of asset risk. Furthermore, we incorporate the possibility of regulators having limited capacity to enforce strict caps on asset risk by introducing a lower bound on volatility. This level reflects the scenario where banks face constraints on their ability to liquidate risky assets and convert them to risk-free assets.

By incorporating these elements into the Merton model, our framework provides a dynamic representation of the interplay between leverage, asset risk, and regulatory interventions. Our approach represents an intermediate case between two extremes – intervention only happens at debt maturity (Merton 1974, 1978) and liquidation occurs immediately once a certain threshold is breached (Black and Cox 1976). We show how implementing this model can help to assess bank stability by measuring risk signals more accurately, such as the cost of deposit insurance and the credit spread of its debt.

In the standard contingent claim models when there is a decline in the value of assets and leverage increases it leads to a higher probability of default. In our model, the regulatory requirement forces banks to change the portfolio of assets in order to reduce volatility when leverage is high. This reduces the probability of a large further change in asset value. If the regulator acts decisively, the reduction can be substantial. Our dynamic volatility model implies a lower probability of severe distress after an initial negative shock and therefore a significantly lower LGD (Loss Given Default) due to lower volatility when the bank is at risk of having insufficient capital. Thus, for an initial unrestricted level of volatility, the ex-ante default probability, credit spread and cost of deposit insurance are substantially reduced in comparison to the standard case of constant volatility. In turn, when targeting a specific level of default probability, our model implies a higher level of initial leverage. We provide illustrative numerical examples based on realistic parameters and present sensitivity analysis to changes in leverage, volatility, and the speed of regulatory intervention.

The remainder of the paper is organized as follows. Section 2 presents the structure of assets and liabilities, Section 3 defines the regulatory policy and the resulting dynamic nature of volatility. Section 4 discusses the qualitative results of our model and in Section 5 we quantify the effect of intervention speeds on risk using a numerical example; Section 6 concludes.

2. The structure of liabilities

We consider a hypothetical bank with asset value denoted by V_0. The bank is funded by equity with a market value of S_0 and zero-coupon deposits (debt) with a face value of F, time to maturity T, and market value B_0. Default occurs at debt maturity, T, if the value of assets, V_T, is lower than the face value of the debt, F. If default occurs, depositors receive the payment based on the residual assets V_T. Deposits are guaranteed by the government or one of its agencies (e.g., the FDIC); if there is a need, an additional payment of $(F - V_T)$ is made by the guarantor. Otherwise, the debt is fully paid by the bank. The payoff to the bank's stockholders and deposit insurance, D_T, at debt maturity can be written respectively as follow:

$$S_T = \max(V_T - \mathrm{F}, 0)\,, \qquad D_T = \max(\mathrm{F} - V_T, 0).$$

3. Bank assets with dynamic volatility

While the payoffs at maturity are identical to the classical Merton model (1974, 1977 and 1978), the stochastic process of the bank's assets and the regulatory intervention policy are adjusted to the state of the bank. Since the bank is a regulated institution, the supervisor conducts periodic audits at the end each period $i = 1, \dots, T$. As in Merton (1978) and Merton and Bodie (1992), audits are periodic due to surveillance costs. While bank's assets have constant volatility between audits, at the end of each period the regulator sets an upper limit on volatility if the quasi leverage ratio drops below a certain threshold. The usage of such a measure is in line with the Basel II guidelines where a market-based model is used to limit and to manage credit risk.[1]

As in Merton (1977) regulators use the quasi leverage ratio, defined as the ratio between the discounted face value of debt and the value of assets, V_t, as a supervisory risk indicator:

$$LR_t = Fe^{-r(T-t)}/V_t. \tag{1}$$

The level of volatility is constant in the periods between audits, and the value of the bank's assets between two periodic audit events follows a geometric Brownian motion process:

$$dV_t = \mu_V V_t dt + \sigma_i V_t dW_V \tag{2}$$

where μ_V is the instantaneous expected return, σ_i is the volatility during the audit period and dW_V is a standard Wiener process.

At each audit, the regulator measures the quasi-leverage ratio and volatility is set to:

$$\sigma_i = \sigma_{Min} + (\sigma_0 - \sigma_{Min}) e^{-\mathbb{1}_{\{LR_t > H\}}(LR_t - H)\omega} \tag{3}$$

where $\mathbb{1}_\psi$ is an indicator function of the event ψ. If leverage is below the threshold H, which is determined by the regulator, the bank keeps operating under the predetermined initial level of volatility σ_o. If leverage is above the threshold H, the regulator intervenes and imposes a lower level of volatility, which depends on the deviation from the threshold, i.e. the severity of the

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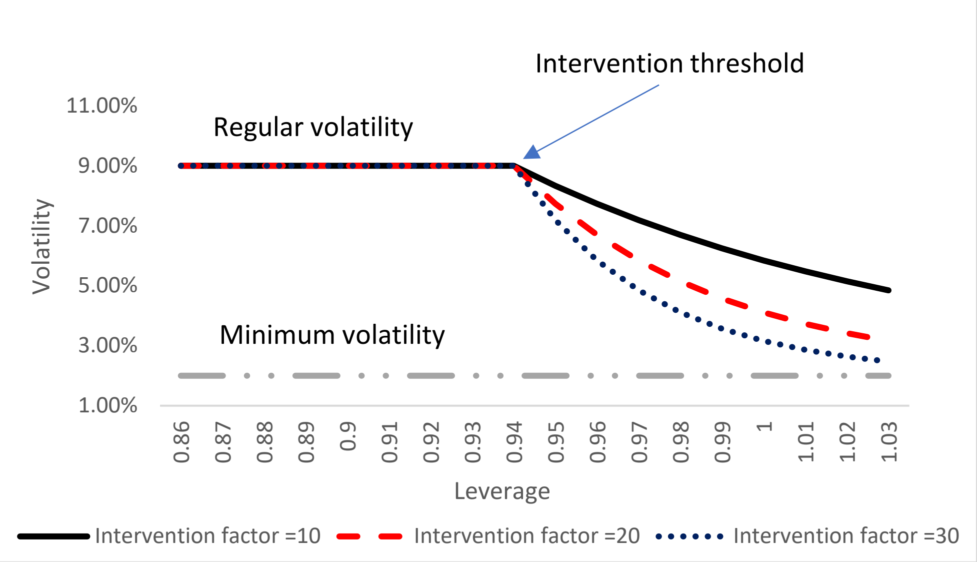
[1] Bond, Goldstein and Prescott (2010) criticized the usage of market-based measures by showing that if agents use market prices when deciding on corrective actions, prices are adjusted to reflect this use and potentially become less revealing.

financial distress the bank is in. The size of the volatility reduction depends on how decisively the regulatory acts. It depends on the will and the ability of the regulator and is captured by the parameter ω – the larger the decisiveness of the intervention, the larger the reduction in volatility. However, the ability of the bank to replace risky assets with less risky or risk-free assets is limited and therefore there is a lower bound to the level of asset risk in our model, σ_{Min}.

Figure 1 illustrates the regulatory policy by showing volatility as a function of leverage at the audit. When leverage is low, there are no regulatory restrictions and the level of volatility is set by management. The bank chooses investments with the highest NPV regardless of the level of risk (Jensen and Smith 2000) and the volatility is equal to its initial level σ_o. When leverage increases and lies above the cutoff H, the regulator demands a reduction of the volatility. The higher the leverage, the bigger the required reduction, subject to the lower bound σ_{Min} and the regulatory intervention factor ω.

Figure 1: Illustration of the bank's risk dynamics versus the quasi-leverage ratio



The figure shows the post-audit level of asset risk for different quasi leverage ratios at the time of the audit. The leverage intervention threshold $H = 0.94$ and the initial level of volatility equals $\sigma_o = 9\%$. When leverage is above the threshold volatility decreases exponentially with leverage until it reaches a level of $\sigma_{Min} = 2\%$. The strength of the intervention of the regulator ω is set equal to 10, 20 or 30.

4. Qualitative implications of regulation of financial intermediaries

Our model has implications for various measures of bank risk, as compared to the standard fixed-volatility model. As long as the leverage ratio is below the threshold, the proposed model gives the same dynamics as for unregulated firm (with constant volatility). However due to the regulatory requirements, as soon as the leverage ratio exceeds the threshold, the volatility declines gradually and the value of assets becomes less volatile (risky). The level of volatility does not go to zero but subsequent changes in asset value are smaller. This has implications for several important financial indicators. Taking the perspective of a bank close to the regulatory threshold at time zero, here are a few of the main features of our approach:

1. Bank debt is less risky than for a similar non-financial firm. The value of debt is therefore higher and the credit spread smaller. In addition, the debt rating is higher than if it had been issued by an unregulated firm.
2. The value of equity (which is a call option in Merton's model) is lower due to the lower expected future volatility of the assets in time of distress.
3. PD (probability of default) and LGD (loss given default) for financial institutions are lower than comparable non-financial firms.
4. Financial firms will be able to have a much higher leverage ratio and still keep a relatively high rating.
5. The transition matrix for financial institutions will be different from non-financial institutions. Specifically, the probability of lower ratings changing will be reduced relative to non-financial firms.

Specific values of parameters in our model need to be calibrated to each regulator and market. We use some numerical examples below to demonstrate the key features of the proposed model. Because of space limitations we do not provide examples for all of the predictions above. Instead, we focus on the effect of risk regulation on the cost of deposit insurance which is a major measure for banks' risk.

5. Dynamic asset risk in practice

We now quantify the effect of dynamic asset risk regulation on the cost of deposit insurance. The effects on the other risk indictors (default probability, credit spread) are similarly large but are not reported to save space (available upon request). For our numerical example we choose realistic parameters which are based on other papers that have recently calibrated models of financial institutions (Hilscher et al, 2021; Mehran and Rosenberg 2008). We then calculate the cost of deposit insurance using a Monte Carlo simulation to generate a stochastic process with dynamic volatility.

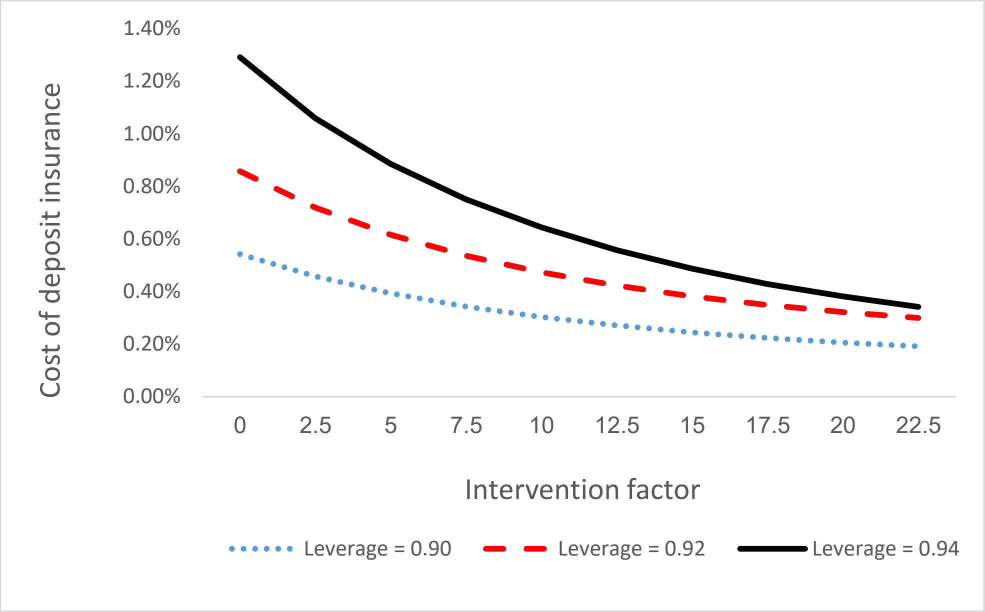
We use the following parameters: leverage is 0.9, 0.92, or 0.94; time to maturity is one year, T=1; the risk-free rate is r=3%. We assume an initial level of asset volatility of 9%; the intervention level of leverage is $H = 0.94$ and the minimum level of volatility is $\sigma_{Min} = 2\%$. The main regulatory parameter that sets the nature of the regulatory intervention – how the post-audit volatility depends on leverage – is the intervention factor ω.

Figure 2 presents the cost of deposit insurance for different levels of initial leverage and intervention factors ω. When the limit on asset risk is ineffective and volatility is not adjusted after periodic audits ($\omega = 0$), the model converges to the classical Merton (1974, 1977) model where intervention occurs only at debt maturity. Thus, when leverage is 0.92 and volatility is fixed at a level of 9% the cost of deposit insurance is 0.88%. However, when the limit is imposed and $\omega = 20$, the cost of deposit insurance drops to 0.47% of the face value of deposits. The effect of regulation is even larger when leverage is equal to 0.94. In this case, the regulated bank, assuming an intervention factor of $\omega = 20$ has a cost of deposit insurance of 0.64%, much lower than the 1.58% implied by the standard model or for an unregulated bank.

We next consider the effect of initial volatility. Figure 3 reports the cost of deposit insurance for different levels of volatility and again across different levels of the assumed intervention factor. If asset volatility is high, the fixed-volatility model implies a cost of 2.3%, which shrinks to 0.9% if $\omega = 20$. At a lower level of initial volatility, 7%, the reduction is smaller, moving from 0.95% to 0.35% of the face value of deposits.

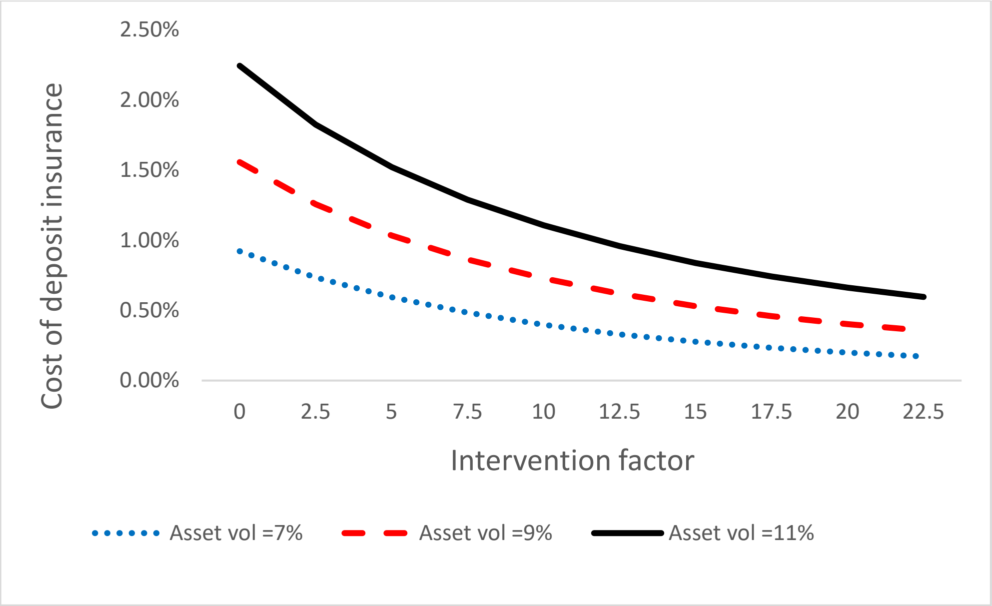
To summarize the effect of dynamic volatility, in Table 1 we report stock value, bond value, cost of deposit insurance, LGD, and credit spread for different intervention speeds. In the case of no intervention, $\omega = 0$, the stock price, LGD and the credit spread are higher, while the bond price is lower as compared to the case of regulatory intervention at various speeds.

Figure 2: Cost of deposit insurance for different initial leverage and intervention factors



The figure shows the cost of deposit insurance as a percentage of the face value of debt for different intervention factors (ω). The intervention threshold and the minimum asset risk are the same as in Figure 1. Initial asset volatility equals 9% and all other parameters are as described in Section 5.

Figure 3: Cost of deposit insurance for different initial volatility and intervention factors



The figure shows the cost of deposit insurance as a percentage of the face value of debt for different intervention factors (ω). Initial asset volatility equals 7%, 9% or 11%. All other parameters are the same as in Figure 2.

Table 1: Effect of regulator intervention factor on bank liabilities, credit spread, and LGD

Intervention factor	**Stock price**	**Bond price**	**Cost of deposit insurance (% of face value)**	**LGD**	**credit spread**
0	8.84	96.65	0.40%	2.0%	0.41%
10	8.71	96.77	0.28%	1.5%	0.28%
20	8.64	96.84	0.20%	1.2%	0.21%
30	8.59	96.89	0.15%	1.0%	0.16%
40	8.56	96.92	0.12%	0.9%	0.13%

The table shows the stock price, bond price (value of deposits not taking into account deposit insurance), deposit insurance, LGD (loss given default), and the credit spread for different intervention factors (ω). Initial asset volatility equals 7%, and leverage is set to 92%. All other parameters are the same as in Figure 3.

6. Conclusion

In this paper we develop a model of a regulated financial institution with asset dynamics that reduces volatility when the leverage ratio increases above a certain threshold. This model has bank-specific predictions on valuation and risk. We assume that the regulator steps in at intermittent audit periods to reduce volatility if leverage breaches a predetermined threshold. This policy is in line with Basel II guidelines and the practice of bank stress tests that was adopted after the 2008 financial crisis by national and international regulators. Consistent with observed empirical patterns, regulated financial institutions have lower default probability, lower overall risk, and can be highly levered. All of these features of banks cannot be easily explained using the standard model and are naturally included in the proposed modification.

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