Online Appendix: “Bank stability and market discipline: The effect of contingent capital on risk taking and default probability”

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Abstract

Section A of the appendix presents the option building blocks used to value bank liabilities. Section B presents the building blocks for the calculation of equity vega. In Section C we discuss equity vega for different capital structures.

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A Option building blocks: Pricing

We provide here the pricing formulas of the five barrier options that serve as building blocks for the valuation of the bank's liabilities. The formulas are derived by Merton (1973) and Rubinstein and Reiner (1991).

Following the Black and Scholes (1973) assumptions, the risk-free interest rate is constant over time and equal to r. The value of assets, denoted by S, is well described under the risk neutral probability by the following stochastic differential equation:

$$\frac{dS}{S} = rdt + \sigma dW$$

where W is a standard Brownian motion, and σ is the instantaneous constant standard deviation of the asset's rate of return. All options expire at time T, have a strike price of K and a barrier level of H.

1. *Down-and-in* call option: If S (the underlying asset) reaches the barrier H, the option becomes a vanilla European call option with strike price K and maturity T. If S does not reach the barrier H the option expires worthless. The value of the option with a barrier that is strictly larger than the strike price is:

$$\begin{aligned} CB^{din}(H,K) &= SN(d_1) - Ke^{-rT}N(d_2) + S\left(\frac{H}{S}\right)^{\left(\frac{2r}{\sigma^2}+1\right)} N(d_5) \\ &\quad -Ke^{-rT}\left(\frac{H}{S}\right)^{\left(\frac{2r}{\sigma^2}-1\right)} N(d_6) - SN(d_3) + Ke^{-rT}N(d_4) \end{aligned}$$

   where $N(.)$ denotes the standard normal cumulative probability distribution function. The value of a *down-and-in* call option with a barrier which is smaller than the strike price is:

$$CB^{din}(H,K) = S\left(\frac{H}{S}\right)^{\left(\frac{2r}{\sigma^2}+1\right)} N(d_7) - Ke^{-rT}\left(\frac{H}{S}\right)^{\left(\frac{2r}{\sigma^2}-1\right)} N(d_8).$$

2. *Down-and-out* call option: If S does not reach the barrier H, the option becomes a vanilla European call option with strike price K and maturity T. If S reaches the barrier H the option expires worthless. The value of the option with a barrier that is strictly larger than the strike price is:

$$CB^{dout}(H,K) = SN(d_3) - Ke^{-rT}N(d_4) - S\left(\frac{H}{S}\right)^{\left(\frac{2r}{\sigma^2}+1\right)} N(d_5) + Ke^{-rT}\left(\frac{H}{S}\right)^{\left(\frac{2r}{\sigma^2}-1\right)} N(d_6).$$

   The value of a *down-and-out* call option with a barrier which is strictly smaller than the strike price is:

$$CB^{dout}(H,K) = SN(d_1) - Ke^{-rT}N(d_2) - S\left(\frac{H}{S}\right)^{\left(\frac{2r}{\sigma^2}+1\right)} N(d_7) + Ke^{-rT}\left(\frac{H}{S}\right)^{\left(\frac{2r}{\sigma^2}-1\right)} N(d_8).$$

3. *Down-and-out* put option: If S does not reach the barrier H, the option becomes a vanilla European put option with strike price K and maturity T. If S reaches the barrier H the option expires worthless. The value of the option with a barrier that is strictly smaller than the strike price is:

$$\begin{aligned}
& PB^{out}(H,K) \\
= \; & SN(d_1) - Ke^{-rT}N(d_2) - S\left(\frac{H}{S}\right)^{\left(\frac{2r}{\sigma^2}+1\right)} N(d_5) + Ke^{-rT}\left(\frac{H}{S}\right)^{\left(\frac{2r}{\sigma^2}-1\right)} N(d_6) \\
& -SN(d_3) + Ke^{-rT}N(d_4) + S\left(\frac{H}{S}\right)^{\left(\frac{2r}{\sigma^2}+1\right)} N(d_7) - Ke^{-rT}\left(\frac{H}{S}\right)^{\left(\frac{2r}{\sigma^2}-1\right)} N(d_8)
\end{aligned}$$

where

$$\begin{aligned}
d_1 &= \frac{\ln\left(\frac{S}{K}\right)+T\left(r+\frac{\sigma^2}{2}\right)}{\sigma\sqrt{T}} & d_2 &= \frac{\ln\left(\frac{S}{K}\right)+T\left(r-\frac{\sigma^2}{2}\right)}{\sigma\sqrt{T}} \\
d_3 &= \frac{\ln\left(\frac{S}{H}\right)+T\left(r+\frac{\sigma^2}{2}\right)}{\sigma\sqrt{T}} & d_4 &= \frac{\ln\left(\frac{S}{H}\right)+T\left(r-\frac{\sigma^2}{2}\right)}{\sigma\sqrt{T}} \\
d_5 &= \frac{\ln\left(\frac{H}{S}\right)+T\left(r+\frac{\sigma^2}{2}\right)}{\sigma\sqrt{T}} & d_6 &= \frac{\ln\left(\frac{H}{S}\right)+T\left(r-\frac{\sigma^2}{2}\right)}{\sigma\sqrt{T}} \\
d_7 &= \frac{\ln\left(\frac{H^2}{SK}\right)+T\left(r+\frac{\sigma^2}{2}\right)}{\sigma\sqrt{T}} & d_8 &= \frac{\ln\left(\frac{H^2}{SK}\right)+T\left(r-\frac{\sigma^2}{2}\right)}{\sigma\sqrt{T}}.
\end{aligned}$$

4. *Down-and-out* digital call option: If S does not reach the barrier H, the option pays one unit of currency at maturity T if the value of assets at maturity is greater than the strike price K. If S reaches the barrier H the option expires worthless. The value of the option with a barrier that is strictly larger than the strike price is:

$$DB^{dout}(H,K) = Ke^{-rT}\left[N(d_4) - \left(\frac{H}{S}\right)^{\left(\frac{2r}{\sigma^2}-1\right)} N(d_6)\right].$$

The value of a *down-and-out* digital call with a barrier which is strictly smaller than the strike price is:

$$DB^{dout}(H,K) = Ke^{-rT}\left[N(d_2) - \left(\frac{H}{S}\right)^{\left(\frac{2r}{\sigma^2}-1\right)} N(d_8)\right].$$

5. *Down-and-in* digital call option (with payoff at touch): If S reaches the barrier H the

option pays one unit of currency at touch. If S does not reach H the option expires worthless. The value of the option is:

$$DB^{din}(H) = \left(\frac{H}{S}\right)^{\frac{2r}{\sigma^2}} N(d_5) + \frac{S}{H} N(d_9)$$

where

$$d_9 = \frac{\ln\left(\frac{H}{S}\right) - T\left(r + \frac{\sigma^2}{2}\right)}{\sigma\sqrt{T}} .$$

B Equity vega building blocks

We present here the sensitivity to asset risk of the four types of options which are used as building blocks for the valuation of the bank's stock.

1. *Down-and-out* call option with strike price which is strictly larger than the barrier (K>H): The derivative w.r.t. volatility is

$$\begin{aligned} \frac{\partial CB^{dout}(H,K)}{\partial\sigma} &= V\sqrt{T}n(d_1) - \left(\frac{H}{V}\right)^{\left(\frac{2}{\sigma^2}+1\right)} V\sqrt{T}n(d_7) \\ &\quad - \left(\frac{4r}{\sigma^3}\right) e^{rT} \ln\left(\frac{V}{H}\right) \left(\frac{H}{V}\right)^{\left(\frac{2r}{\sigma^2}+1\right)} \left[\begin{array}{c} VN(d_7) \\ -\left(\frac{V}{H}\right)^2 Ke^{-rT} N\left(d_7 - \sigma\sqrt{T}\right) \end{array}\right] \end{aligned}$$

where $n(.)$ denotes the standard normal density function. (See Johnson and Tian, 2000).

2. *Down-and-out* call option with strike price which is strictly smaller than the barrier (K<H): The derivative w.r.t. volatility is

$$\begin{aligned} \frac{\partial CB^{dout}(H,K)}{\partial\sigma} &= V\sqrt{T}n(d_3) - \frac{Vd_3 n(d_3)(H-K)}{\sigma H} \\ &\quad - \left(\frac{H}{V}\right)^{\left(\frac{2}{\sigma^2}+1\right)} \left[V\sqrt{T}n(d_5) - \frac{Vd_5 n(d_5)(H-K)}{\sigma H}\right] \\ &\quad - \left(\frac{4r}{\sigma^3}\right) e^{rT} \ln\left(\frac{V}{H}\right) \left(\frac{H}{V}\right)^{\left(\frac{2r}{\sigma^2}+1\right)} \left[\begin{array}{c} VN(d_5) \\ -\left(\frac{V}{H}\right)^2 Ke^{-rT} N\left(d_5 - \sigma\sqrt{T}\right) \end{array}\right]. \end{aligned}$$

3. *Down-and-in* call option: A position which includes both a *down-and-out* barrier call option and a *down-and-in* barrier call option with the same strike price and maturity results in the price of a plain vanilla call option as follows:

$$C(K) = CB^{dout}(H,K) + CB^{din}(H,K).$$

Therefore, the vega of the *down-and-in* call option can be expressed as the difference between the vega of a regular call option and the vega of a *down-and-out* barrier option with the same strike and barrier level and is equal to

$$\frac{\partial CB^{din}\left(H,K\right)}{\partial\sigma}=V\sqrt{T}n\left(d_1\right)-\frac{\partial CB^{dout}\left(H,K\right)}{\partial\sigma}.$$

C Equity vega for different capital structures

C.1 Contingent capital

We show that it is possible to find a conversion ratio so that equity vega is equal to zero. We proceed as follows. If $\alpha=1$, vega is negative and if $\alpha=0$, vega is non-negative. Since vega is continuous in α, it must be the case that a $0\leqslant\alpha\leqslant1$ exists such that vega is zero.

Recalling the value of equity from equation (9, in the paper), if $\alpha=1$ the value of equity is equal to a *down-and-out* call option with a strike price of F^D and a barrier of K^C (which lies above or is equal to the strike price). Such an option has a negative vega (or the vega can be zero).

If $\alpha=0$ then the equity value is equal to the sum of a *down-and-out* and a *down-and-in* call option that have the same barrier, but where the former has a higher strike price. If the two options had the same strike, the position would be identical to that of a plain vanilla call option, and so would have the same vega as such an option. (If the barrier is touched, only the *down-and-in* option remains, if the barrier is not touched, only the *down-and-out* option remains.) Since the *down-and-in* call option has a lower strike price its vega is in fact a little higher since, holding the barrier constant, its vega increases as the strike price decreases. The vega of a plain vanilla call option is positive and so the vega of the combination is positive since it is larger than (or equal to) the vega of a plain vanilla call option with a strike price of the face value of deposits F^D. We now add the short position in the *down-and-in* call option with a strike of F^D and a barrier of K^D (the final term in equation (9, in the paper)). The vega of this option is positive but is always lower than the vega of a corresponding plain vanilla call option with the same strike price. The reason is that its barrier may not be touched so that its value is lower than the corresponding plain vanilla call option. Therefore, the difference between the sum of the first two terms and the third term is positive, which is what we were trying to show. We conclude that an intermediate value of the conversion threshold always exists that results in a zero equity vega.

We also note that equity vega is monotonic in the conversion threshold. The vega of the difference of the *down-and-in* call options is positive (or zero) since the first one has a higher barrier. Therefore, as the conversion threshold decreases from $\alpha=1$, the equity vega increases.

C.2 Equity and deposits

The equity vega of a bank with a capital structure that includes deposits and stock is positive unless the seizing policy parameter γ is very small and leverage is high. Equity vega is equal to the derivative of a *down-and-out* call option with a strike price that is equal to the face value of deposits and a barrier level that is located γ percent below the strike price (setting $F^B=0$ in equation (11, in the paper)). We can show that, since the value of assets is larger than the barrier, the vega is positive as long as $\frac{H^2 e^{rT}}{Ke^{-rT}V} < \frac{N\left(d_7 - \sigma\sqrt{T}\right)}{N(d_7)}$. Assuming that volatility lies between 1% to 9% (the range we consider, Table 2), this inequality holds as long as there are some limits to the seizing power of the regulator (as we assume in our analysis).

Only if γ is very close to zero and leverage is high can stockholders face incentives to reduce risk (and avoid the bank being seized). In this case the marginal increase in upside potential (that results from an increase in asset volatility) is more than offset by the higher likelihood of default, in which case equity holders receive nothing.

C.3 Subordinated debt

For a bank with subordinated debt in its capital structure the motivation to increase risk is always greater than for a bank with only stock and deposits. The position of the stockholder in such a bank can be replicated by a *down-and-out* call option with a barrier which is located γ percent below the face value of deposits (as in the stock and deposit bank, see equation (11, in the paper)) but a strike price that is strictly higher by the face value of subordinated debt (F^B). The vega of such an option increases as the strike price approaches the value of assets. Therefore the vega for the case of subordinated debt is always higher than that for the case of equity and deposits.